ROADEF 2023 (Rennes)

Bilevel Problem with Switching costs: Application to Electricity Pricing

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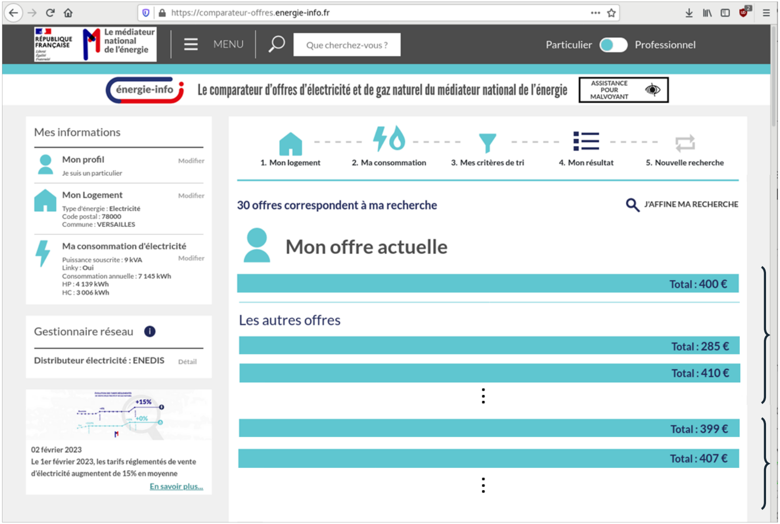
Section 1

Definition of the model

The consumer’ decision *at time* t



Offers of my current provider

Offers of other providers

Figure: Example of price comparison engine (French electricity market)

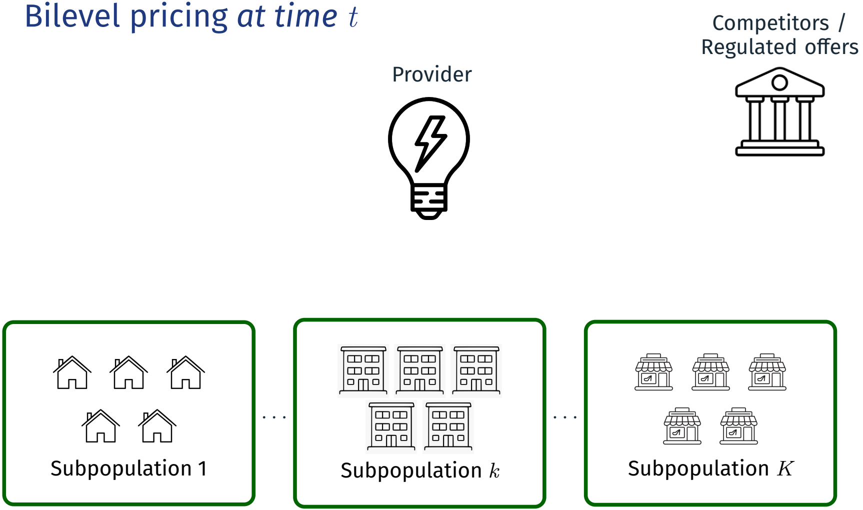
Inertia in consumer reaction

Intuition (Dubé et al., 2010; Horsky and Pavlidis, 2010)

"I switch to a new contract if there is a *sufficient* difference with my current offer."

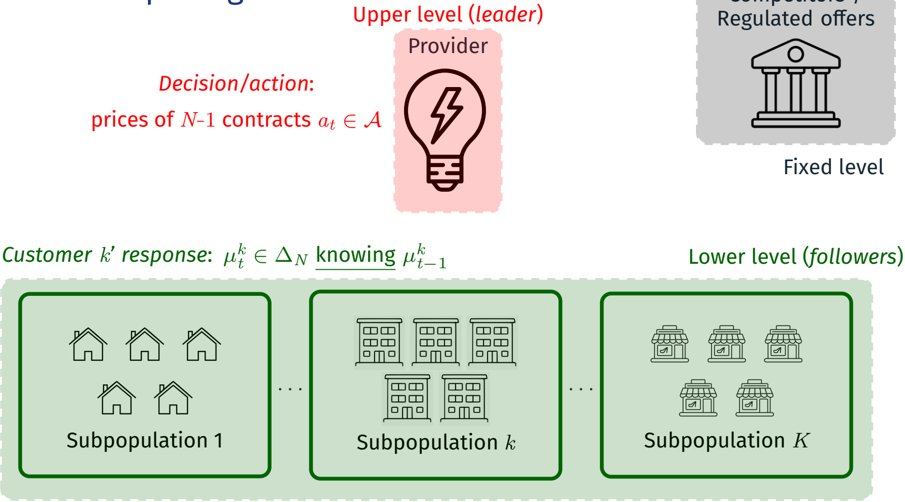
Image from https://www.sketchbubble.com/en/presentation-switching-costs.html

Bilevel pricing *at time* t



Segmentation: Each subpopulation is composed of *similar* consumers (same behavior)
Static competition: Prices of competitors are *given* $\Rightarrow$ *single-leader* problem

Bilevel pricing *at time* t



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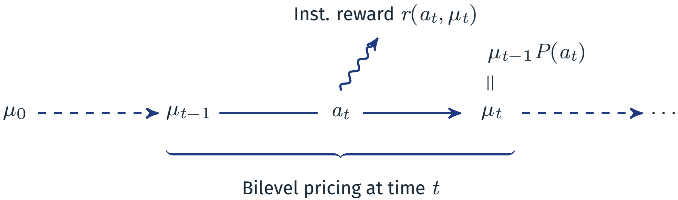
High-level description as lifted MDP[1]



1. *Heterogeneous population*: each cluster $k \in [K]$ represents a proportion ρ_k of the overall pop.
2. *Distribution*: $\mu_t^k \in \Delta_N$ the distribution of the population of cluster k over $[N]$.
3. *Instantaneous reward*: $\theta^k(\cdot)$ denotes the instantaneous *unitary* reward,

   $r : (a_t, \mu_t) \mapsto \sum_{k \in [K]} \rho_k \left\langle \theta^k(a_t), \mu_t^k \right\rangle_N$ ← upper objective at time t

4. *(Linear) Transition*: $\mu_t^k = \mu_{t-1}^k P^k(a_t)$ ← lower decision at time t
5. *Leader's (global) objective* (average long-term reward):

$$g^*(\mu_0) = \sup_{\pi \in \Pi} \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} r(\pi_t(\mu_t), \mu_t) \ . \qquad \text{(AvR)}$$

[1] In a Mean-field context, see e.g. Motte and Pham, 2019

Specification to the Electricity Market context

Given k and an offer $n < N$, we know

- *Reservation price* R^{kn}: max. price that k want to spend on n,
- *Energy consumption* E^{kn}: fixed consumption if k chooses n,
- *Utility* $U^{kn}(a) := R^{kn} - E^{kn}a^n$, where a^n is the price for one unit of n.

The utility of the alternative option is normalized to 0, i.e., $U^{kN} = 0$.

The (linear) unitary reward for the provider is then

$$\theta^{kn}(a) = \underbrace{E^{kn}a^n}_{\text{electricity invoice}} - \underbrace{C^{kn}}_{\text{cost}}, \; n < N, \quad \theta^{kN} = 0 \; .$$

Assumption: The transition probability follows a *logit response*, see e.g. Pavlidis and Ellickson, 2017:

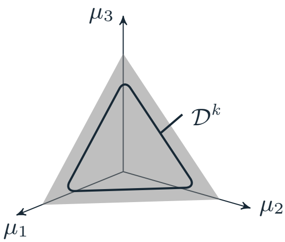
$$[P^k(a)]_{n,m} = \frac{e^{\beta[U^{km}(a)+\gamma^{kn}\mathbb{1}_{m=n}]}}{\sum_{l\in[N]} e^{\beta[U^{kl}(a)+\gamma^{kn}\mathbb{1}_{l=n}]}} > 0 \; ,$$

- γ^{kn} is the cost for segment k to *switch* from contract n to another one,
- β is the intensity of the choice (it can represent a "*rationality* parameter").

Section 2

Ergodic control

Ergodic control



Let $\mathcal{D}^k := \mathrm{vex}\left(\{\mu^k P^k(a) \mid a \in \mathcal{A}, \mu^k \in \Delta_N\}\right)$,
and $\mathcal{D} = \bigtimes_{k\in[K]} \mathcal{D}^k$.

Lemma

$\mathcal{D}^k \subseteq \mathrm{relint}\ \Delta_N^K$.
Moreover, for $t \geq 1$, $\mu_t \in \mathcal{D}$ for any policy $\pi \in \Pi$.

For $v : \Delta_N^K \to \mathbb{R}$, the *Bellman operator* $\mathcal{B}$ is

$$\mathcal{B}\, v\,(\mu) = \max_{a\in\mathcal{A}}\{r(x,\mu) + v(\mu P(a))\} \ .$$

Theorem

The *ergodic eigenproblem*

$$g\,\mathbb{1}_{\mathcal{D}} + h = \mathcal{B}\, h$$

admits a solution $g^* \in \mathbb{R}$ and h^* Lipschitz and convex on $\mathcal{D}$.
Moreover, g^* satisfies (AvR), and $a^*(\cdot) \in \arg\max \mathcal{B}\, h^*$ defines an *optimal policy*.

Deterministic MDP without controllability – the most degenerate case

	Time	Transitions	Assumption
Schweitzer, 1985	discrete	stochastic	unichain[2]
Biswas, 2015	discrete	stochastic	Doeblin / minorization[3]
Mallet-Paret and Nussbaum, 2002	discrete	deterministic	quasi-compactness
Fathi, 2010	continuous	deterministic	controlability[4]
Zavidovique, 2012	discrete	deterministic	controlability
Calvez et al., 2014	continuous	deterministic	contraction of the dynamics $(A2)$
This work	discrete	deterministic	contraction of the dynamics $(A2)$

Standard unichain/Doeblin type conditions entail that the eigenvector is *unique*, up to an additive constant, this is *no longer true* in our case.

[2] the Markov Chain induced by any deterministic stationary policy consists of a single recurrent class plus a –possibly empty– set of transient states (i.e., there exists a subset of states that are visited infinitely often with probability 1 independently of the starting state)

[3] for all state s, action a and measurable subset B of the state space, $P(B|x, a) \geq \epsilon\mu(B)$

[4] for every pair of states (s, s'), there exists an action a making s' accessible from s

Ergodic control – Sketch of the proof (existence)

We use a contraction argument directly on the dynamics (*not on* the Bellman Operator):
Let d_H be the Hilbert's projective metric defined as

$$d_H(u, v) = \max_{1 \le i,j \le n} \log \left(\frac{u_i}{v_i} \frac{v_j}{u_j} \right) \quad .$$

$(\mathcal{D}, d_H)$ is a complete metric space.

Birkhoff theorem

Every matrix $Q \gg 0$ is a contraction in Hilbert's projective metric, i.e.,

$$\forall \mu, \nu \in (\mathbb{R}^N_{>0}), \; d_H(\mu Q, \nu Q) \le \kappa_Q d_H(\mu, \nu) \quad ,$$

where $\kappa_Q := \tanh\left(\mathrm{Diam}_H(Q) \,/\, 4\right) < 1$.

We then use the method of *vanishing discount approach* (Lions et al., 1987):

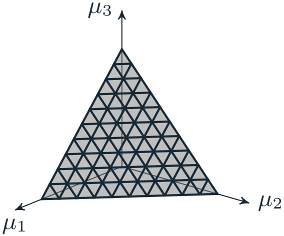
$\rightarrow$ the family of α-discounted objective function $\left(V_\alpha(\cdot)\right)_\alpha$ is *equi-Lipschitz*, which entails the existence of the eigenvector by a *compactness* argument.

Section 3

Algorithms

Relative Value Iteration with Krasnoselskii-Mann damping

- Regular grid Σ of the simplex Δ_N^K,
- Bellman Operator $\mathcal{B}^\Sigma$ using Freudenthal triangulation (Lovejoy, 1991).



Algorithm RVI with Mann-type iterates

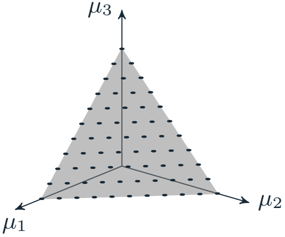
Require: Σ, $\mathcal{B}^\Sigma$, $\hat{h}_0$
1: $v_{max} \leftarrow -\infty$
2: Initialize $\hat{h} = \hat{h}_0$, $\hat{h}'(\mu) = \mathcal{B}^\Sigma \, \hat{h}$
3: **while** $\text{sp}(\hat{h}' - \hat{h}) > \epsilon$ **do**
4: $\quad \hat{h} \leftarrow (\, \hat{h}' - \max\{\hat{h}'\}e + \hat{h}\,)/2$
5: $\quad \hat{h}'(\hat{\mu}) \leftarrow (\mathcal{B}^\Sigma \, \hat{h})(\hat{\mu})$ for all $\hat{\mu} \in \Sigma$
6: **end while**
7: $\hat{g} \leftarrow (\, \max(\hat{h}' - \hat{h}) + \min(\hat{h}' - \hat{h})\,)/2$
8: **return** $\hat{g}, \hat{h}$

Proposition (Gaubert and Stott, 2020)

Convergence time of RVI = $O(\epsilon^{-2})$

Policy Iteration

- Regular grid Σ of the simplex Δ_N^K,
- Bellman Operator $\mathcal{B}^{\Sigma}$ using semi-lagrangian discretization.



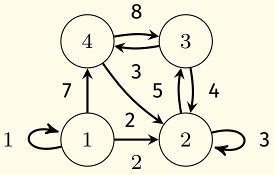
On-the-fly generation of transitions, refining (C.-Terrasson et al., 1998).

↪ solve the spectral problem
$$\max_{1 \le j \le n} (A_{ij} + x_j) = \lambda + x_i \ .$$

↪ the transition is *decomposed* on each segment

Example

$$A = \begin{bmatrix} 1 & 2 & \mathbb{0} & 7 \\ \mathbb{0} & 3 & 5 & \mathbb{0} \\ \mathbb{0} & 4 & \mathbb{0} & 3 \\ \mathbb{0} & 2 & 8 & \mathbb{0} \end{bmatrix}$$



Proposition

PI has *finite* time convergence

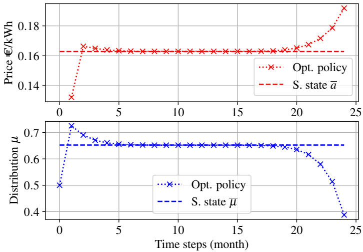
Section 4

Application to electricity pricing

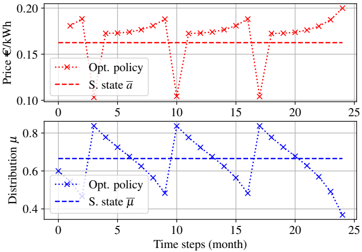
Impact of switching costs γ on toy model

"Turnpike" like strategy:
Attraction to a steady-state

Cyclic strategy:
A promotion is periodically applied



(a) Optimal finite horizon trajectory (provider action and customer distribution) for *low* switching cost.



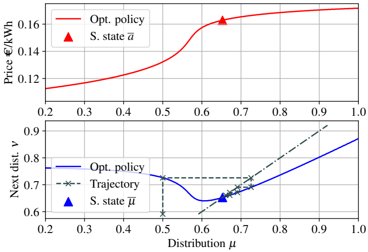
(b) Optimal finite horizon trajectory (provider action and customer distribution) for *high* switching cost.

$\hookrightarrow$ Confirms *optimality of periodic promotions*, already observed in Economics, see e.g. Horsky and Pavlidis, 2010.

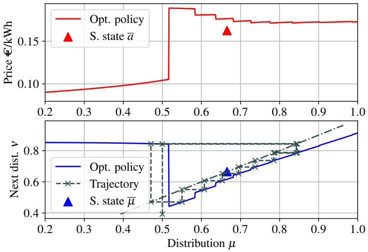
Impact of switching costs γ on toy model

<u>"Turnpike" like strategy</u>:
Attraction to a steady-state

<u>Cyclic strategy</u>:
A promotion is periodically applied



(a) Optimal decision for the long-run average reward (provider action and next customer distribution)



(b) Optimal decision for the long-run average reward (provider action and next customer distribution)

$\hookrightarrow$ Confirms *optimality of periodic promotions*, already observed in Economics, see e.g. Horsky and Pavlidis, 2010.

Conclusion and Perspectives

Conclusion

- ◇ Resolution of a MDP with bilevel transition via an eigenproblem representation
- ◇ Refinement of Policy Iteration for Heterogeneous populations
- ◇ Application to electricity pricing, and highlight of the switching cost's impact

Perspectives

- ◇ Conditions for the convergence to a steady-state
- ◇ Study of other lower-level behaviors (non logit-based)

$\rightsquigarrow$ All this work was published here:

Jacquet, Q., van Ackooij, W., Alasseur, C., & Gaubert, S. (2022). Ergodic control of a heterogeneous population and application to electricity pricing.
2022 IEEE 61st Conference on Decision and Control (CDC)

Thank you for your attention !

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References

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Numerical results

Instance[5]	(node, arcs)	RVI-KM	PI[6]	This work[7]
$K = 2, N = 2$ $\delta_\mu = 1/50$	$(7.4\ 10^5, 6.9\ 10^8)$	7h 15Mo	390s 13Go	70s 103Mo

[5] K: segments, N: contracts, δ_μ: discretization's precision (for each dimension)
[6] Cochet-Terrasson et al., 1998
[7] Each method ran on a 10 threads on a laptop i7-1065G7 CPU@1.30GHz.

Steady-states

Theorem

Given a constant *action a, the distribution sequence $\left(\mu_t^k\right)_t$ converges to $\overline{\mu}^k(a)$, defined as*

$$\overline{\mu}^{kn}(a) = \frac{\eta^{kn}(a)\mu_L^{kn}(a)}{\sum_{l\in[N]}\eta^{kl}(a)\mu_L^{kl}(a)} \quad . \tag{1}$$

where $\eta^{kn}(a) := 1 + \left[e^{\beta\gamma^{kn}} - 1\right]\mu_L^{kn}(a)$, and

$$\mu_L^{kn} = e^{\beta\, U^{kn}(a)} \Big/ \sum_{l\in[N]} e^{\beta\, U^{kl}(a)} \quad . \tag{2}$$

As a consequence, the optimal steady-state can be found by solving the *static* problem

$$\overline{g} = \max_{a\in\mathcal{A}} r(a, \overline{\mu}(a)) \quad . \tag{3}$$