Dagstuhl Seminar

Bilevel optimization for the retail electricity market

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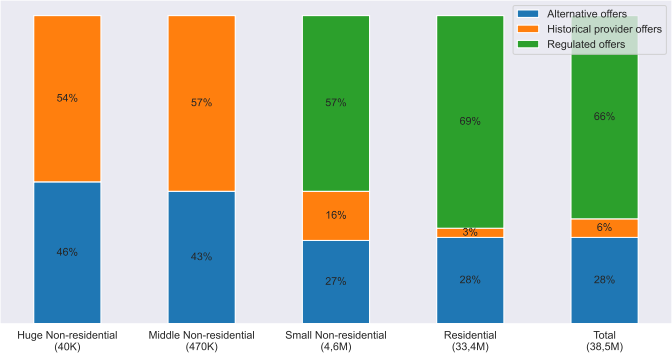
Introduction

A wide variety of offers...

◇ Since 2007, French electricity market is open to competition :

Market Offers Company freely determines the prices	*Regulated Offers* Fixed prices



A WIDE VARIETY OF OFFERS...

- Since 2007, French electricity market is open to competition :

Market Offers
Company freely determines the prices

Regulated Offers
Fixed prices

- Contracts structure:

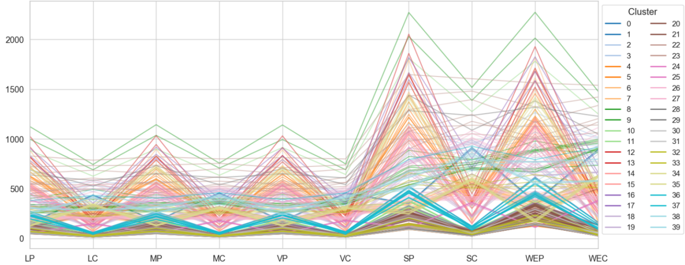
W contracts

	Baseload version	Peak/Off-peak version
Variable portion (€/kWh)	unique price	peak price
		off-peak price
Fixed portion (€)	power	power

H attributes (2 or 3)

… AND A WIDE VARIETY OF CUSTOMERS

- Load curves[1] of customers reflect different consumption behaviors:



- Consumption preferences change

  Digital ("Digiwatt"), Green ("Vert Electrique")
  Self-consumption (solar panel, batteries, ...)

Assumption: The population can be aggregated into S customers segments.

[1] We use simulated load curves, from SMACH.

Challenge

Issue

How to determine fair prices to attract/keep customers while secure a sufficient profit ?

Leader-follower game:

- First player (*leader*) decides
- Second player (*follower*) reacts

Sketch of the modelization

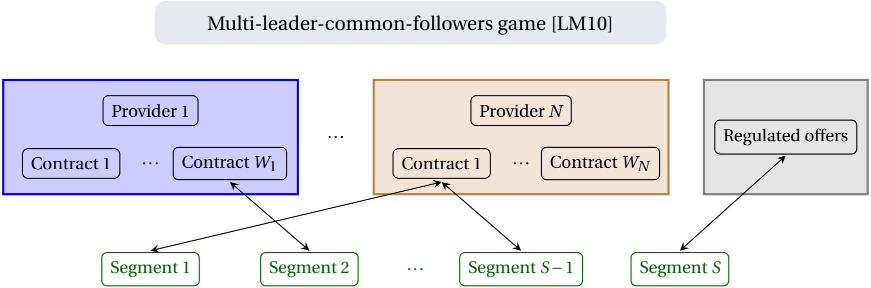


Figure: Representation of the problem

- Nash equilibrium at upper level
- *Envy-Free* : no limitation on the maximum number of customers able to purchase the same contract

FOCUS ON STATIC COMPETITION

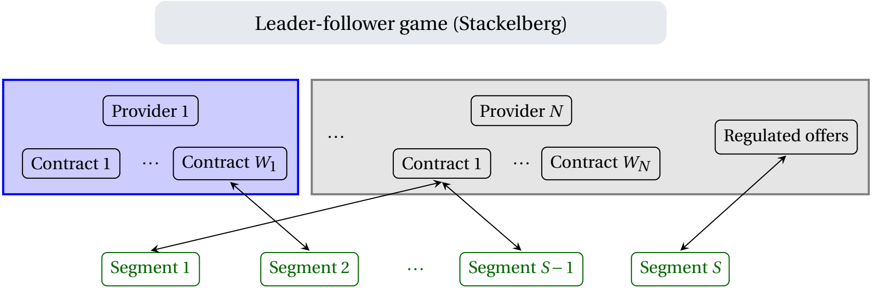


Figure: Representation of the problem

- ~~*Nash equilibrium at upper level*~~ → static competition
- *Envy-Free* : no limitation on the maximum number of customers able to purchase the same contract

Focus on static competition

Leader-follower game (Stackelberg)

General formulation of *Bilevel problems* [Dem+15]

$$\begin{cases} \text{“}\min_{x}\text{”} \quad F(x,y) & \leftarrow \text{“Upper level”} \\ \quad \text{s.t} \quad x \in X & \\ \quad\quad y \in \Psi(x) := \operatorname*{Argmin}_{y} \left\{ f(x,y)\,;\, g(x,y) \leq 0 \right\} & \leftarrow \text{“Lower level”} \end{cases}$$

- x is called “Upper variable”, controlled by the leader
- y is called “Lower variable”, controlled by the follower

Complexity results

Linear Bilevel problems are NP-Hard [Jer85].

LEADER-FOLLOWER GAMES

[Jac+21]

DETERMINISTIC MODEL

Notations:

- $[S] := \{1 \dots S\}$ customers segments,
- $[W]$ contracts of the leader,
- $[H]$ attributes per contract

Variables:

- x_w^h price of attribute h and contract w,
- $\mu_{sw} = \begin{cases} 1 \text{ if segment } s \text{ chooses } w, \\ 0 \text{ otherwise.} \end{cases}$

Data:

- C_{sw} cost to supply s if he chooses w,
- R_{sw} *reservation price* of s for contract w,
- Customer invoice is a *linear form* of the prices

$$\theta_{sw}(x) := \langle E_{sw}, x_w \rangle_H$$

Deterministic bilevel problem

$$\begin{cases} \boxed{\max_{x \in X, \mu^*} \sum_{s \in [S]} \rho_s \langle \theta_s(x) - C_s, \mu_s^* \rangle_W} \rightarrow \text{leader pb} \\ \text{s.t.} \quad \boxed{\mu^* \in \operatorname*{argmin}_{\mu \in (\Delta_{W+1})^S} \left\{ \sum_{s \in [S]} \langle \theta_s(x) - R_s, \mu_s \rangle_W \right\}} \\ \quad \hookrightarrow \text{follower pb} \end{cases}$$

Profit function

$$\pi(x) := \sum_{s \in [S]} \rho_s \langle \theta_s(x) - C_s, \mu_s^*(x) \rangle_W \ ,$$

where $\mu^*(\cdot)$ is the optimal follower response.

KKT transformation

The follower problem is linear, and can be replaced by KKT conditions:

$$\max_{x \in X, \mu, \eta} \quad \sum_{s \in [S]} \rho_s \eta_s + \rho_s \langle R_s - C_s, \mu_s \rangle_W$$

$$\text{s.t.} \quad \boxed{\begin{aligned} & 0 \leq \mu_{sw} \perp \theta_{sw}(x) - R_{sw} - \eta_s \geq 0, \ \forall s, w \\ & 0 \leq \mu_{s0} \perp \eta_s \leq 0, \ \forall s \\ & \mu_s \in \Delta_{W+1}, \ \forall s \end{aligned}}$$

This leads to a *Linear Program under Complementarity Constraints* (LPCC).

Usually, we replace the complementarity constraints by Big-M constraints
$\rightsquigarrow$ MILP formulation, generalization of [STM11; Fer+16].

Price complex and instability

One customer ($S = 1$), 2 contracts ($W = 2$)

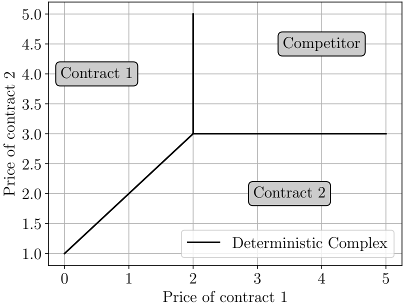


Figure: Response of follower in the space of prices

(developped in [BK19; Eyt18])

Five customers ($S = 5$), 1 contract ($W = 1$)

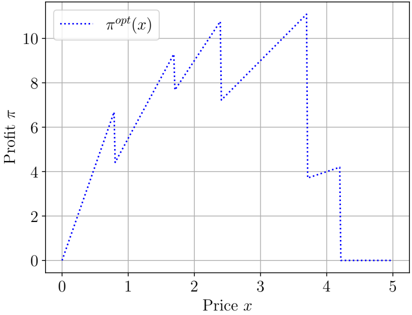


Figure: Instability in the profit function

(developped in [GMS15])

Proposition [Jac+21]

In the general case, the optimal profit is achieved at a discontinuity.

Logit regularization

Multinomial Logit model

$$\begin{cases} \max_{x \in X, \mu^*} \sum_{s \in [S]} \rho_s \langle \theta_s(x) - C_s, \mu_s^* \rangle_W \\ \text{s.t.} \quad \mu^* \in \operatorname*{argmin}_{\mu \in (\Delta_{W+1})^S} \left\{ \begin{aligned} &\sum_{s \in [S]} \langle \theta_s(x) - R_s, \mu_s \rangle_W \\ &+ \frac{1}{\beta} \langle \log(\mu_s), \mu_s \rangle_{W+1} \end{aligned} \right\} \end{cases}$$

$$\rightsquigarrow \mu_{sw}^*(x) = \frac{e^{-\beta(\theta_{sw}(x) - R_{sw})}}{1 + \sum_{w' \in [W]} e^{-\beta(\theta_{sw'}(x) - R_{sw'})}}$$

$\Rightarrow \mu_s^* \in \operatorname{Int} \Delta_{W+1}$, no polyhedral complex

Five customers ($S = 5$), 1 contract ($W = 1$)

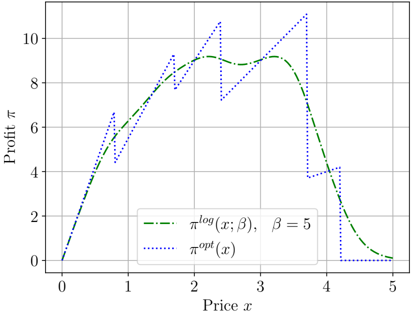


Figure: Logit regularization

(developped in [GMS15; Li+19])

Proposition [Li+19]

For a heterogeneous population ($S > 1$), π is in general non-concave.

Quadratic regularization (1)

Multinomial Logit model

$$\begin{cases} \max_{x \in X, \mu^*} \sum_{s \in [S]} \rho_s \langle \theta_s(x) - C_s, \mu_s^* \rangle_W \\ \text{s.t.} \quad \mu^* \in \underset{\mu \in (\Delta_{W+1})^S}{\operatorname{argmin}} \begin{Bmatrix} \sum_{s \in [S]} \langle \theta_s(x) - R_s, \mu_s \rangle_W \\ + \frac{1}{\beta} \langle \log(\mu_s), \mu_s \rangle_{W+1} \end{Bmatrix} \end{cases}$$

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Quadratic model

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$$\rightsquigarrow \mu^*_s(x) = \operatorname{Proj}_{\Delta_{W+1}} \left(\frac{\beta}{2} (R_s - \theta_s(x)) \right)$$

$\hookrightarrow$ powerfull algorithm for projection on the simplex, see [Con16]

Quadratic regularization (2)

One customer ($S = 1$), 2 contracts ($W = 2$)

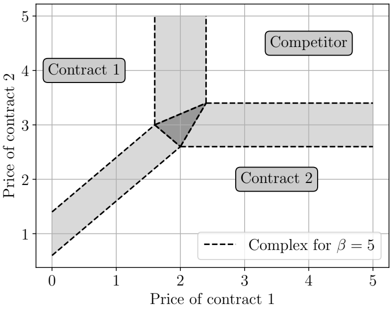

Figure: Response of follower in the space of prices

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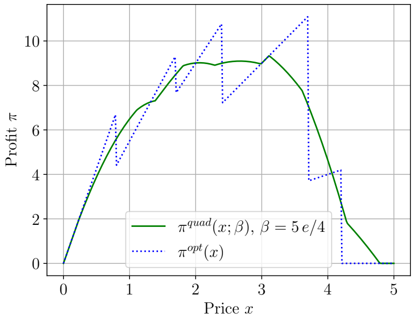


Figure: Quadratic regularization

Theorem [Jac+21]

The profit π is *continuous*. Moreover, it is *concave* on each cell of the polyhedral complex.

Customers' response as a polyhedral complex

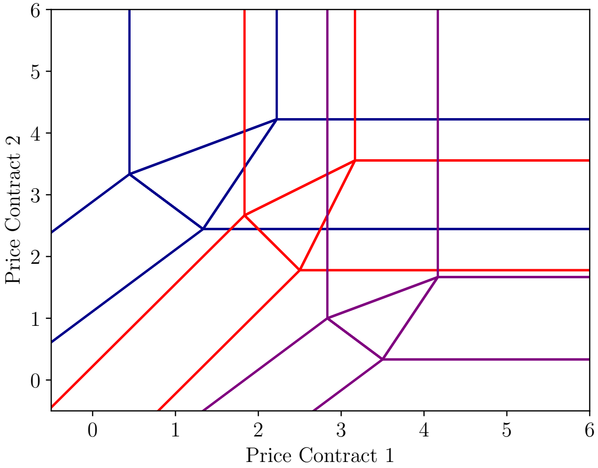
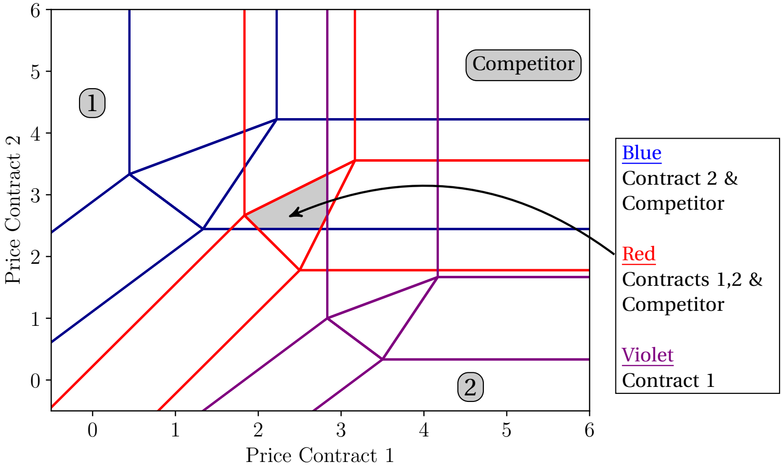


Figure: Example with $S = 3$ segments and $W = 2$ contracts

Customers' response as a polyhedral complex



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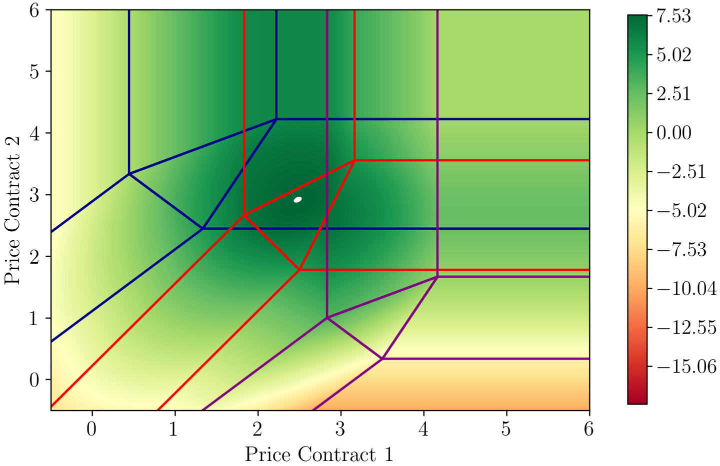
Design of a heuristic – On an example



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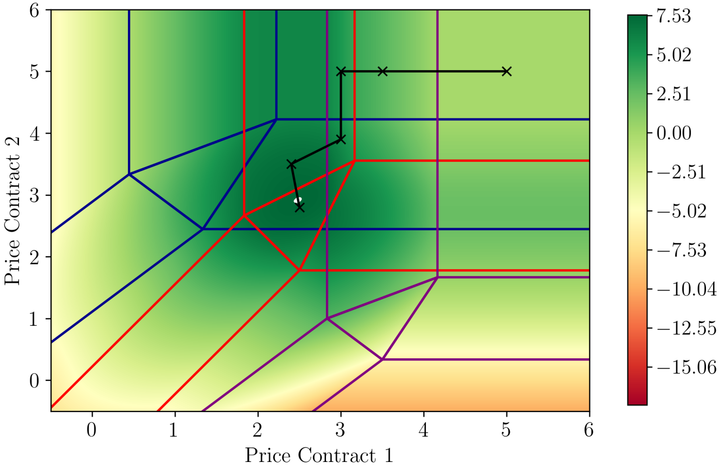

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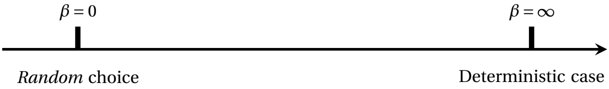
Impact of the regularization intensity



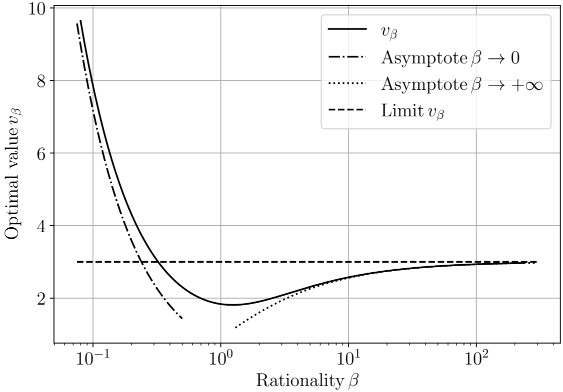


Figure: Optimal value as a function of β

Demand Elasticity

(Ongoing research)

And if consumers *adapt* their consumption to prices ?

Intuition

"If the electricity is too costly, I will reduce my consumption."

$\hookrightarrow$ Isoelastic utility function of the electricty demand (CRRA):

$$U_s : E \in \mathbb{R}^H \mapsto \sum_{h \in [H]} \alpha_s^h \frac{(E^h)^\eta}{\eta}, \ \eta \in \underbrace{]-\infty, 0[}_{\text{residential}} \cup \underbrace{]0,1[}_{\text{industrial}} \quad .$$

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- The customer *not only* decides the contract, but also maximizes

$$U_s^* : x \in \mathbb{R}^H \mapsto \max_{E \in \mathbb{R}^H} \{U_s(E) - \langle x, E \rangle_H\} \quad .$$

The optimal energy consumption is $\mathcal{E}_s^h(x^h) = \left(\frac{\alpha_s^h}{x^h}\right)^{\frac{1}{1-\eta}}$.

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The optimal energy consumption is $\mathscr{E}_s^h(x^h) = \left(\frac{\alpha_s^h}{x^h}\right)^{\frac{1}{1-\eta}}$.

→ The invoice is now a *nonlinear* function:

$$\cancel{\theta_{sw}(x_w) := \langle \check{E}_{sw}, x_w \rangle_H} \to \Theta_s(x_w) := \langle \mathscr{E}_s(x_w), x_w \rangle_H$$

Remark: We recover the inelastic case for $\eta \to -\infty$.

DISTORTION OF THE POLYHEDRAL COMPLEX

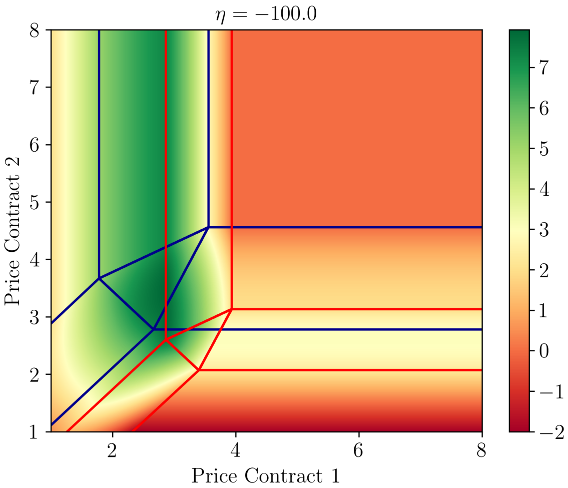
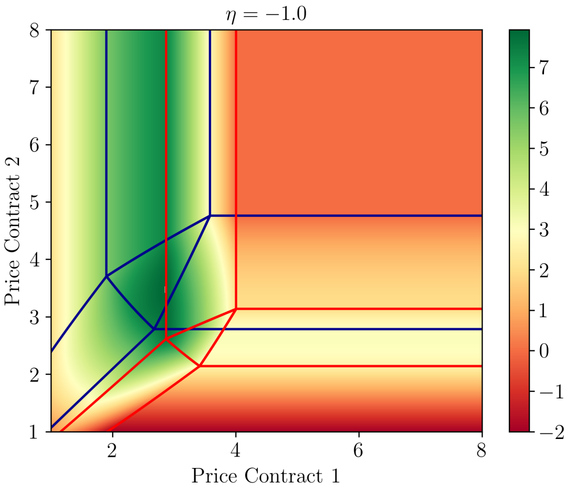


Figure: Example with $S = 2$ segments and $W = 2$ contracts

Distortion of the Polyhedral Complex



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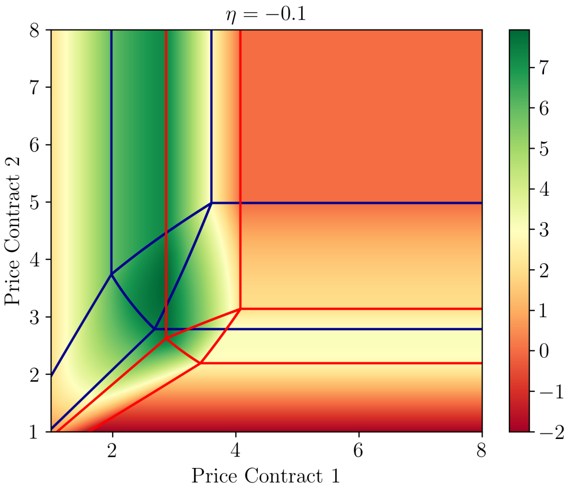

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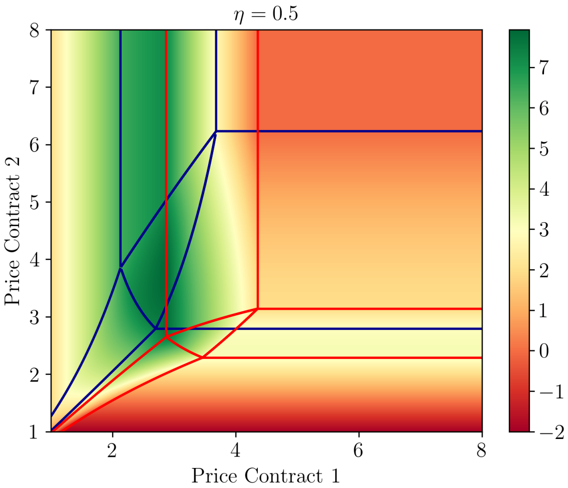


Figure: Example with $S = 2$ segments and $W = 2$ contracts

Retrieving a Polyhedral Complex (first order cost)

Proposition

Suppose that the price constraints are of the following form

$$X = \mathscr{O}_{(\underline{x},\overline{x},\kappa)}(P) := \left\{ x_w^h \in [\underline{x}_w^h, \overline{x}_w^h] \;\middle|\; x_w^h \leq \kappa_w^h x_{w'}^{h'} \text{ for } (w,h) \leq_P (w',h') \right\} ,$$

where P is a partial order set. Then, the bilevel problem

$$\max_{x \in X, \mu^*} \sum_{s \in [S]} \rho_s \langle \Theta_s(x) - C_s, \mu_s \rangle_W$$

$$\text{s.t. } \mu^* \in \underset{\mu \in (\Delta_{W+1})^S}{\operatorname{argmin}} \left\{ \sum_{s \in [S]} \langle \Theta_s(x_w) - R_s, \mu_s \rangle_W \right\} ,$$

can be equivalently defined using variables $z_w^h := (x_w^h)^{-\frac{\eta}{1-\eta}}$

$$X \dashrightarrow Z$$

nonlinear price complex | polyhedral complex

CONSUMPTION-DEPENDENT COSTS

The retailer cost is not constant anymore, but depends on the total consumption:

$$\cancel{\max_{x\in X,\mu^*} \sum_{s\in[S]} \rho_s \langle \Theta_s(x) - C_s, \mu_s^* \rangle_W} \to \max_{x\in X,\mu^*} \sum_{s\in[S]} \rho_s \langle \Theta_s(x), \mu_s^* \rangle_W - C \underbrace{\left(\sum_{s\in[S]} \rho_s \sum_{w\in[W]} \mathscr{E}_s(x_w) \mu_{sw}^* \right)}_{:=\mathscr{E}^{\text{tot}}(x,\mu^*)\ \text{(total consumption)}}$$

with $C(\cdot)$ a convex nondecreasing function.

Proposition

In the Z space,

- the energy consumption $z_w \mapsto \mathscr{E}_s(z_w)$ is always *convex*,
- the total energy $z \mapsto \mathscr{E}^{\text{tot}}(z, \mu^*(z))$ is *convex* on each cell for a sufficiently large regularization intensity β^{-1}.

DYNAMIC EXTENSION

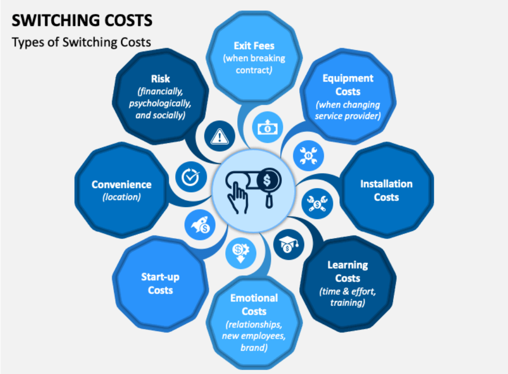
[Jac+22]

AND IF CONSUMERS *do not immediately* REACT ?

Intuition

"I switch to a new contract if there is a *sufficient* difference with my current offer."

This notion is known is Economics:
$\hookrightarrow$ Customers have *switching costs* (imperfect market), see e.g. [DHR10; HP10]



MARKOVIAN DECISION PROCESS

Modelization as a *Markovian Decision Process* (MDP)

$$\mu_{t+1} = \mu_t P(x_t),$$

where $P(x_t)$ is the transition matrix,
obtained by solving the *lower* problem knowing the *upper* decision x_t at time t.

We choose a *logit* transition

$$P(x_t) = diag\left(\{P(x_t)_s\}_{s\in[S]}\right), \quad [P_s(x_t)]_{(v,w)} = \frac{e^{\beta(R_{sw}-\theta_{sw}(x_t))+\gamma_{sv}\mathbb{1}_{(w=v)}}}{1+\sum_{w'\in[W]} e^{\beta(R_{sw'}-\theta_{sw'}(x_t))+\gamma_{sv}\mathbb{1}_{(w'=v)}}}$$

$\rightsquigarrow$ γ_{sw} is the switching cost the customer s would pay if he switches to another offer.

- The previous (static) model is recover when $\gamma \equiv 0$.
- $P(x) \gg 0$ for all x, and we can define $\mathscr{D}$ such that

$$\mu_t \in \mathscr{D} \subset \text{relint}\left(\Delta_W^S\right), \; t \geq 1 \;\; .$$

Ergodic control

For a policy $\pi = \{\pi_t\}_{t\geq 1}$, $x_t = \pi_t(\mu_t)$ is the action taken by the controller at t.
Now, we aim to maximize the *average long-term reward*, i.e.,

$$g^* = \sup_{\pi\in\Pi} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} r(\pi_t(\mu_t), \mu_t) \ , \tag{1}$$

where $r(\cdot,\cdot)$ is the objective defined in the static model.

For any function $v : \Delta_W^S \to \mathbb{R}$, the Bellman operator $\mathscr{B}$ is defined as

$$\mathscr{B}\, v(\mu) = \max_{x\in X}\{r(x,\mu) + v(\mu P(x))\} \ .$$

Theorem [Jac+22]

Assume that $x \mapsto P_s(x)$ is continuous and $P_s(x) \gg 0$ for all x and $s \in [S]$. Then, the ergodic eigenproblem

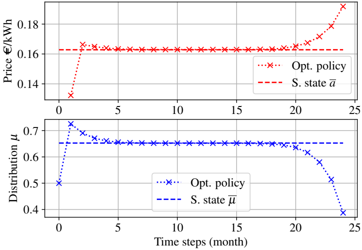
$$g\mathbb{1}_{\mathscr{D}} + h = \mathscr{B}\, h$$

admits a solution $g^* \in \mathbb{R}$ and h^* Lipschitz and convex on $\mathscr{D}$.
Moreover, g^* satisfies (1), and a maximizer $x^*(\cdot) \in \arg\max \mathscr{B}\, h^*$ defines an optimal policy for the average gain problem.

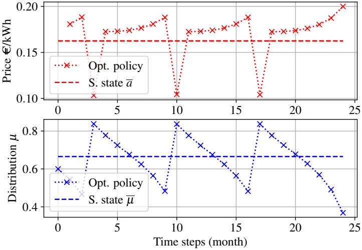
IMPACT OF SWITCHING COSTS ON TOY MODEL

“Turnpike” like strategy:
Attraction to a steady-state

Cyclic strategy:
A promotion is periodically applied



(a) Optimal finite horizon trajectory (provider action and customer distribution) for *low* switching cost.



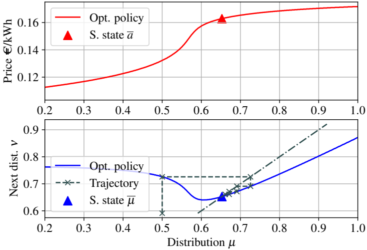
(b) Optimal finite horizon trajectory (provider action and customer distribution) for *high* switching cost.

$\hookrightarrow$ Phenomenon already mentioned in Economics, see e.g. [HP10].

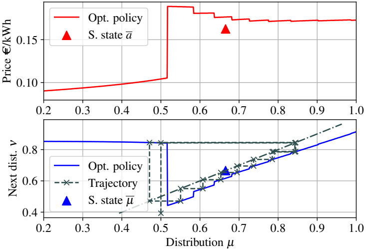
Impact of switching costs on toy model

<u>"Turnpike" like strategy</u>:
Attraction to a steady-state

<u>Cyclic strategy</u>:
A promotion is periodically applied



(a) Optimal decision for the long-run average reward (provider action and next customer distribution)



(b) Optimal decision for the long-run average reward (provider action and next customer distribution)

$\hookrightarrow$ Phenomenon already mentioned in Economics, see e.g. [HP10].

Perspectives

Future works

- Analyze of turnpike property for the dynamic extension
- Definition of continuous-time model
- Competition at the upper level (between leaders)

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THANK YOU FOR YOUR ATTENTION !